parabola with reference to its axis are $\dfrac{a\cos^3\theta}{\cos^2\frac{1}{2}\theta}$ and $\dfrac{2a\sin\theta\cos^2\theta}{\cos^2\frac{1}{2}\theta}$, and its parameter $\dfrac{a\sin^2\theta\cos\theta}{\cos^2\frac{1}{2}\theta}$.

Denoting by x and y the current co-ordinates of this curve with reference to the axis of the given parabola, and the focus as origin. we find from the above

$$x=\frac{a\cos\theta+a\sin^2\theta\cos\theta-a\cos^3\theta}{\cos^2\frac{1}{2}\theta}=\frac{a\sin 2\theta\sin\theta}{\cos^2\frac{1}{2}\theta},$$

$$y=a\frac{\sin\theta-\sin 2\theta\cos\theta}{\cos^2\frac{1}{2}\theta}=-\frac{a\sin\theta\cos 2\theta}{\cos^2\frac{1}{2}\theta}.$$

By eliminating θ we get $(x^2+y^2-4a^2)^2(x^2+y^2)=y^2(x^2+y^2+4a^2)^2$. Introducing polar co-ordinates ρ and ϕ, we get the simpler equation

$$\rho=\pm 2a\cot(45^\circ-\tfrac{1}{2}\phi) \text{ and } \rho=\pm 2a\tan(45^\circ-\tfrac{1}{2}\phi).$$

Also solved by G. B. M. Zerr.

AVERAGE AND PROBABILITY.

160. Proposed by J. F. LAWRENCE, A. M., Stillwater, Oklahoma.

Two points are taken at random in a triangle, the line joining them dividing the triangle into two portions. Find the mean value of that portion containing the center of gravity.

Solution by HENRY HEATON, Belfield, N. D.

The triangle may be considered equilateral (see Williamson's *Integral Calculus*, p. 355). Put $\triangle$ =area of equilateral triangle whose side$=a$. Let P be one of the points, $PD=y$ and $BD=x$. Let EPF be the line through the two points, and put $\angle BFP=\theta$ and $\angle BEP=\phi$. Then $\phi=\frac{2}{3}\pi-\theta$, $PF=y\operatorname{cosec}\theta$ and $DF=y\cot\theta$. The area of the elemental triangle $PFF'=\frac{1}{2}y^2\operatorname{cosec}^2\theta d\theta$. We will suppose the second point to be confined to this elemental triangle. Put $BF=z$. Then $z=x+y\cot\theta$, $BE=z\sin\theta\operatorname{cosec}\phi$, and area of triangle $EBF=\dfrac{\triangle z^2}{a^2}\sin\theta\operatorname{cosec}\phi$.

If $\theta<\frac{1}{6}\pi$ and F is confined to the line BC the area of the portion of the triangle containing the center of gravity is $\dfrac{\triangle}{a^2}(a^2-z^2\sin\theta\operatorname{cosec}\theta)$. The limits of z for this are $\frac{2}{3}y\sqrt{3}\sin\phi\operatorname{cosec}\theta$ and a. Those of y are 0 and $\frac{1}{2}a\sqrt{3}\sin\theta\operatorname{cosec}\phi$. If $\theta>\frac{1}{6}\pi$ and $<\frac{1}{3}\pi$, the line EF passes through O when $z=\dfrac{a}{3}(1+\sin\phi\operatorname{cosec}\theta)$.

If z is less than this, y is less than $\frac{a}{6}\sqrt{3}(1+\sin\theta\,\mathrm{cosec}\phi)$.

If $z>\frac{a}{3}(1+\sin\phi\,\mathrm{cosec}\theta)$ and $y<\frac{a}{6}\sqrt{3}(1+\sin\theta\,\mathrm{cosec}\phi)$ the limits of z are $\frac{a}{3}(1+\sin\phi\,\mathrm{cosec}\theta)$ and a.

$$\Delta=\triangle\Big[\int_0^{\frac{1}{6}\pi}\tfrac{1}{32}\sin\theta\,\mathrm{cosec}^3\phi-\tfrac{1}{48}\sin^2\theta\,\mathrm{cosec}^4\phi+\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi}\tfrac{1}{48}\sin^2\theta\,\mathrm{cosec}^4\phi-\frac{1}{2\cdot 3^7}$$
$$\times(\sin^2\theta\,\mathrm{cosec}^4\phi+\sin^2\phi\,\mathrm{cosec}^2\theta)+\frac{1}{2^5\cdot 3^6}(\sin\theta\,\mathrm{cosec}^3\phi+\sin\phi\,\mathrm{cosec}^3\theta+\frac{41}{2^4,3^7}$$
$$\times\,\mathrm{cosec}\theta\,\mathrm{cosec}\phi\Big]d\theta\div\tfrac{1}{32}\int_0^{\frac{1}{3}\pi}\sin\theta\,\mathrm{cosec}^3\phi\,d\theta=\frac{\triangle}{3^6}(470+\tfrac{41}{3}\log 2)=.6997\triangle.$$

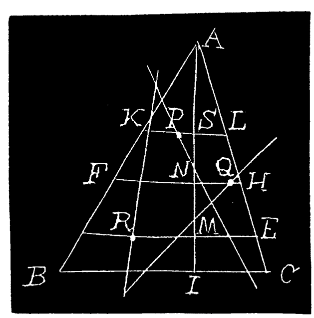
This is problem 76, p. 513, Williamson's *Integral Calculus.*

170. **Proposed by LON C. WALKER, A. M., Santa Barbara, California.**

Find the area of a triangle formed by drawing a line at random through each of three points taken at random within a given triangle.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va.

Let ABC be the given triangle; P, Q, R the random points; $AI=h$, $BI=d$, $CI=e$, $d+e=a$, $AM=u$, $MR=v$, $AS=w$, $SP=z$, $AN=m$, $NQ=n$.



$y-v=r(x-u)$, the line through R...(1),
$y-z=s(x-w)$, the line through P...(2),
$y-n=t(x-m)$, the line through Q...(3),
where $r=\tan\theta$, $s=\tan\phi$, $t=\tan\psi$.

The intersection of (1) and (2) is given by

$$x_1=\frac{ru-ws+z-v}{r-s},\quad y_1=\frac{rsu-rsw+rz-sv}{r-s}.$$

The intersection of (1) and (3) is given by

$$x_2=\frac{ru-mt+n-v}{r-t},\quad y_2=\frac{rtu-mrt+rn-tv}{r-t}.$$

The intersection of (2) and (3) is given by

$$x_3=\frac{sw-mt+n-z}{s-t},\quad y_3=\frac{stw-mst+sn-tz}{s-t}.$$

Area of triangle $=\frac{1}{2}(x_2y_1-x_1y_2+x_3y_2-x_2y_3+x_1y_3-x_3y_1)$
$=\frac{1}{2}[(ur-v)(s-t)+(ws-z)(t-r)+(mt-n)(r-s)]^2/(r-s)(r-t)(s-t)$
$=A/B$.

The limits of u are 0 and h; of w, 0 and u; of m, w and u; of v, $-du/h=v_1$ and $eu/h=v_2$; of z, $-dw/h=w_1$ and $ew/h=w_2$; of n, $-dm/h=m_1$ and $em/h=m_2$. The number of ways the three points can be taken on the surface of the triangle is $\frac{1}{6}(\frac{1}{2}ah)^3=\frac{1}{48}a^3h^3$.